Determination of deviatoric stress tensors based on inversion of calcite twin data from experimentally deformed monophasic samples. Part II. Axial and triaxial stress experiments

Ph. Laurent*a,*, H. Kernb, O. Lacombec

aLaboratoire de Tectonique et Géophysique, Université de Montpellier II, Place E. Bataillon, 34095 Montpellier Cedex, France
bMineralogisch–Petrographisches Institut der Universität Kiel, Kiel, Germany
cLaboratoire de Tectonique Quantitative, ESA 70-72, CNRS, Université Pierre et Marie Curie, 75252 Paris Cedex 05, France

Received 7 January 2000; accepted 3 July 2000

Abstract

Three samples of a bioclastic bajocian limestone from Burgundy (France) were experimentally deformed under axial and triaxial conditions at 200 and 300°C. In each sample, a series of three mutually orthogonal thin sections were studied using a U stage. The intracrystalline deformation of the calcite grains involved mainly twinning on the e planes. Precise measurement of the twin lamellae combined with the Etchecopar inverse method (1984) allowed the calculation of the four parameters of the reduced stress tensor responsible for the twinning. As the principal differential stress values of the experimental deviatoric stress tensors were known, the critical resolved shear stress (CRSS) values for twinning on the e planes were then determined in each case: at 200°C and 4.8% strain, τs is close to 15 MPa; at 300°C and at 8.7 and 9.4% strain, the calculated τs values are close to 10.1 and 13.4 MPa, respectively. When compared with the data published by Turner et al. (1954), Turner and Heard (1965) and Lacombe and Laurent (1996), our results confirm that the CRSS values are more or less dependent on both the temperature and the bulk amount of strain. Consequences for the estimation of the actual principal differential stress values (σ1 − σ3) are described, particularly for very weakly deformed cover rocks in the forelands of mountain chains. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: triaxial tests; calcite; twins; shear stress

1. Introduction

During the last two decades, several studies have dealt with the characterization of present or ancient stress states in cratonic areas and in the foreland of mountain chains (e.g. Engelder and Geiser, 1980; Zoback et al., 1989; Zoback, 1992; Craddock et al., 1993; Arthaud and Laurent, 1995; Lacombe et al., 1996; Van Der Pluijm et al., 1997). In sedimentary rocks of the shallow crust, carbonate rocks are probably the most useful for studying brittle deformation and associated stresses. This is partly due to the efficiency of the pressure-solution process in carbonate rocks and partly to the ability of calcite crystals to deform easily at low temperature by twinning on the e planes (01.2) (Baumhauer in Friedel (1926)). Although these two mechanisms and faulting both contribute to the bulk strain rate, calcite twin analysis can be considered to provide a good estimate of the peak of actual differential stresses (Rowe and Rutter, 1990; Lacombe and Laurent, 1992).
Following on from Lacombe and Laurent (1996), we tackled two key points related to the determination of the magnitudes of the differential stresses using calcite twin inverse analyses in carbonate rocks. (1) The critical resolved shear stress (CRSS) value $\tau_s$ for twinning on the $e$ planes at low temperatures. Using inverse methods, this parameter is necessary to compute the principal differential stress $s_1$, $s_2$, $s_3$ of the deviatoric stress tensor (DST) responsible for the observed twinning. (2) The determination of the stress ellipsoid shape ratio (SESR) $\Phi = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$, when the state of stress is actually triaxial (plane strain conditions). This second parameter is necessary to compute the differential stress values $s_2$ and $s_1$: up to now, these two parameters have been poorly constrained by experimental work under low temperature conditions, although they are essential when considering brittle deformation.

2. Tectonic analysis of calcite twin lamellae

Fig. 1 summarizes the geometry of calcite twinning on the $e$ planes. For more details, see Cahn (1954), Carter and Raleigh (1969), Friedman (1967) and Barber and Wenk (1979). At low temperature, the $e$
twin lamellae are thin and straight (Burkhard, 1993) and may be compared to reverse microfaults when the C axis of the calcite crystal [0001] is oriented vertically (Fig. 1b). Then, the direction, the sense of shear and the shear strain associated with twinning are all constrained by the calcite crystal.

After the pioneering work of Turner (1953), several methods for determining stress or strain from calcite twin lamellae were proposed. A recent review exists on this topic (Burkhard, 1993), so, these methods will not be recalled here. For stress determinations, one must distinguish three basically different approaches.

In the first method, only the principal axes of the incremental strain tensor, i.e. the $P$ and $T$ axes (see discussion in Marrett and Peacock, 1999) are determined (e.g. Turner, 1953, 1962). This method is theoretically justified for monophase deformation if the stress is axial. Numerous applications to experimental and geological deformation have demonstrated the efficiency of Turner’s method (e.g. Friedman and Stearns, 1971; Friedman et al., 1976).

In the second method, only the absolute magnitude of the principal differential stress $(\sigma_1 - \sigma_3)$ is determined (e.g. Friedman and Heard, 1974; Jamison and Spang, 1976; Schmid, 1982; Rowe and Rutter, 1990). This is an advantage of calcite twin lamellae analysis as the magnitude of differential stresses $(\sigma_i - \sigma_j)$ in the Earth’s crust is of central importance for earth scientists. Quantification of the principal differential stress $(\sigma_1 - \sigma_3)$ is made possible by the use of different experimental data from calcite monocrystals or polycrystals such as the twin/grain size dependence (Schmid, 1982), the twinning incidence (percentage of twinned grains with a given grain size), the twin density or volume (Rowe and Rutter, 1990), or a constant threshold value $\tau_s$ equal to the CRSS for twinning on the $e$ planes. For low temperature deformation, the accepted $\tau_s$ value is 10 MPa (e.g. Turner and Heard, 1965; Jamison and Spang, 1976; Tullis, 1980). Nevertheless, it is well known that twinning is more or less dependent on grain size, temperature, and stress heterogeneity (Turner et al., 1954; Olsson, 1974; Spiers, 1979; Tullis, 1980; Schmid, 1982; Rowe and Rutter, 1990; Wheeler, 1991; Newman, 1994; De Bresser and Spiers, 1997). Ferrill (1998) recently used the Jamison and Spang (1976) and Rowe and Rutter (1990) methods to estimate the principal differential stress magnitudes for limestones deformed experimentally and naturally at low temperatures. Overestimation by a factor of up to 20 was obtained and Ferrill concluded that the use of a constant $\tau_s$ value is a “key weakness” of these methods.

Lastly, other methods involve the calculation of the whole DST. The DST is defined by five parameters that may be described as the three principal stress orientations and two values of the differential stresses $(\sigma_i - \sigma_j)$ (Laurent et al., 1981, 1990). Application of these methods depends upon several assumptions: (1) The homogeneous state of stress at the scale of the thin section; i.e. the five parameters of the DST are constant on this scale. (2) Coaxial deformation on the same scale. (3) Low strain (a few percent) at low temperature. These conditions require that the dominant intracrystalline deformation mechanism is twinning on the $e$ planes (Turner et al., 1954; De Bresser and Spiers, 1993, 1997), and no rotation of grains is possible. (4) Samples are made of euhedral calcite grains with no crystallographic preferred orientation. (5) Mechanical twinning is independent of normal stress. Twinning takes place on a given $e$ plane, if and only if the resolved shear stress along the twinning direction $|\epsilon_i - \tau_j|$, in the positive sense (Fig. 1c) is greater than a positive value defined as the CRSS for twinning on the $e$ planes. The validity of these assumptions was discussed by Wenk et al. (1983), Burkhard (1993) and Lacombe and Laurent (1996).


The inverse Etchecopar method (Etchecopar, 1984; Laurent, 1984; Lacombe and Laurent, 1996) is very similar to that used to determine the reduced stress tensor (RST) associated with striated faults sets (Etchecopar et al., 1981). The RST are defined by only four parameters (Angelier, 1984, 1989) which are the three principal stress orientations and the stress ellipsoid shape ratio (SESR) $\Phi = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$. Basically, for striated fault analyses, the use of the RST for calcite twins is justified, primarily for two reasons: first, it was shown that the four parameters of the RST are sufficient to determine uniquely the direction and sense of shear on a given plane (Wallace, 1951; Bott, 1959). Second, the calculation process is...
made easier with only four parameters. We point out that the RST is not a deviatoric stress tensor, as the principal stress values are, respectively: $\sigma_1$, $\sigma_2$, and $\sigma_3$. A simple calculation leads to the deviatoric part of the RST. Thus, it is possible to determine the actual DST by introducing a scaling parameter, namely the CRSS for twinning $\tau_s$, which is supposedly constant for all the calcite grains. The actual differential stress values $\tau_i$ are then proportional to $\tau_s$.

This method makes use of the whole measured $e$ planes data set (twinned and untwinned planes), without any separation prior to the stress inversion (Nemcock et al., 1999). We restate this method briefly only for monophase deformation (for more details, see Etchecopar, 1984). For polyphase deformation, the procedure is the same as that used for striated faults set analysis (see Etchecopar et al., 1981).

The problem is to find an RST which satisfies both the following conditions: (1) For each twin plane, the applied resolved shear stress $\tau_i$ is positive, as defined by Turner (1953) see Fig. 1, and greater than or equal to a critical positive value $\tau_a$. As $(\sigma_1 - \sigma_3)$ is scaled to unity by the RST, the $\tau_i$ values lie somewhere between $+1/2$ and $-1/2$. Then, the $\tau_a$ value corresponds to the actual CRSS value; $\tau_s$ divided by $(\sigma_1 - \sigma_3)$; it lies somewhere between 0 and $+1/2$ (Fig. 2). (2) For each untwinned plane, $\tau_i$ must be lower than $\tau_a$.

First, a great number of RSTs (500 or more) are defined using randomly chosen parameters. The application of all these RSTs to the twin data set allows the calculation of the $\tau_i$ values acting on each $e$ plane. Then, for each RST, all the $e$ planes are classified according to their $\tau_i$ values (Fig. 2). The ideal solution, if it exists, is that for which all the $e$ planes (twinned and untwinned) are classified as shown in Fig. 2a. Obviously, natural samples never fit the theoretical limits and Fig. 2b shows a more realistic classification of the $e$ planes when natural deformation is analyzed.

Thus, four situations may be considered: (1) the compatible twinned planes for which $\tau_i \geq \tau_a$ are on the right side of the diagram in Fig. 2b labeled with $\times$; (2) the incompatible twinned planes, for which $\tau_i < \tau_a$ is labeled with $\otimes$; (3) the compatible untwinned planes for which $\tau_i < \tau_a$ is labeled with $+$; (4) the incompatible untwinned planes for which $\tau_i > \tau_a$ is labeled with $\oplus$. This last set of $e$ planes is very important for the calculation of the RST, because the penalization function $F$ of Etchecopar (1984) which is used to define the best solution, is calculated as the sum of the $(\tau_i - \tau_a)$ values for all the $n$ incompatible untwinned planes

$$F = \sum_{i=1}^{n} (\tau_i - \tau_a)$$

For the ideal case of Fig. 2a, $F$ is equal to 0. For actual

---

Fig. 2. Schematic classification of the $e$ planes, with respect to the applied resolved shear stress $\tau_i$, by the Etchecopar inverse method. Six crystals are considered $(18 e$ planes); $9 e$ planes $(50\%)$ are twinned. The applied resolved shear stress $\tau_i$ increases to the right of the figure, from $-1/2$ to $+1/2$. $\tau_a$ is the lowest resolved shear stress applied on the compatible twinned planes. ($\times$) Compatible twinned planes; ($+$) compatible untwinned planes; ($\otimes$) incompatible twinned planes; ($\oplus$) incompatible untwinned planes. (a) Theoretical situation: $100\%$ of twinned planes are compatible with the RST. Then, the function of penalization $F$ is equal to 0. (b) Example of actual situation: 6 twinned planes $(67\%)$ are compatible with the RST; 2 untwinned planes $(22\%)$ are incompatible with the RST. The function of penalization $F$ is equal to $(\tau_i - \tau_a) + (\tau_2 - \tau_a)$. 

cases, the $F$ values are positive and depend on the applied RSTs. For the example shown in Fig. 2b, $F = (\tau_1 - \tau_2) + (\tau_2 - \tau_3)$. The RST for which $F$ is minimized, yields a first estimate of the best solution, generally quite close to it (see below).

The second step of the Etchecopar method is an optimization process and a refinement of the solution, the same as for striated faults sets analyses. This is done by using a fast nonlinear optimizing procedure, which proceeds by steps in the parameter space. For details, see Etchecopar et al. (1981).

Obviously, the RST calculation must be independent of the incompatible twinned planes. Therefore, the Etchecopar inverse method requires a parameter $P$ which represents the percentage of compatible twins with respect to the whole measured twinned planes set. Because this $P$ value is unknown, several trials are carried out.

### 4. Previous results

To date, the Etchecopar inverse method has been applied to experimental and mainly to natural deformations (e.g. Larroque and Laurent, 1988; Tourneret and Laurent, 1990; Lacombe et al., 1991, 1992, 1993, 1996; Lacombe and Laurent, 1992). Three points emerge from a critical review of this work:

(1) The principal stress orientations are relatively well defined, even when the deformation is polyphased (e.g. Lacombe et al., 1990a,b, 1992, 1996).

(2) The inverse determination of the SESR is more difficult to appreciate for two main reasons: First, the previous experimental work used (Laurent, 1984; Lacombe and Laurent, 1996) always dealt with axial deformation, with $\Phi = 0$. Second, for applications to geological deformations, it is rather risky to compare the results obtained with calcite twins inverse methods to those obtained with striated faults inverse methods (see discussion in Lacombe et al., 1990b, 1992).

(3) The principal differential stress values $(\sigma_1 - \sigma_3)$ were always calculated for a constant threshold $\tau = 10$ MPa. Generally, acceptable results were obtained with this assumption (e.g. Lacombe and Laurent, 1996, Table 1) but for one sample of Carrara marble in this last paper, an overestimation of 440% was calculated. This anomalous value was attributed

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Experimental conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>B21</td>
<td>B31</td>
</tr>
<tr>
<td>Strain ellipsoid</td>
<td>Axial: $\varepsilon_2 = \varepsilon_3$</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>201 ± 1</td>
</tr>
<tr>
<td>Confining pressure $\sigma_1$ (MPa)</td>
<td>100–102</td>
</tr>
<tr>
<td>Isotropic pressure (MPa)</td>
<td>100–185</td>
</tr>
<tr>
<td>Final $\sigma_1$ (MPa)</td>
<td>350</td>
</tr>
<tr>
<td>Final $\sigma_2$ (MPa)</td>
<td>102</td>
</tr>
<tr>
<td>Maximum of $(\sigma_1 - \sigma_3)$</td>
<td>248</td>
</tr>
<tr>
<td>SESR $\Phi$</td>
<td>0</td>
</tr>
<tr>
<td>Final linear strain $\varepsilon_1$ (%)</td>
<td>−4.63</td>
</tr>
<tr>
<td>Final linear strain $\varepsilon_2$ (%)</td>
<td>1.77</td>
</tr>
<tr>
<td>Final linear strain $\varepsilon_3$ (%)</td>
<td>1.84</td>
</tr>
<tr>
<td>Maximum of $\Delta V/V$ (%)</td>
<td>−1.4</td>
</tr>
<tr>
<td>Strain rate ($10^{-6}$ s$^{-1}$)</td>
<td>1.8</td>
</tr>
</tbody>
</table>
to too few untwinned planes to constrain the calculation of the DST. This point will be discussed later in the section concerning the sample B21.

5. Sampling and geological data

For our analysis, we chose a homogeneous poorly deformed sparitic limestone, without any shape or C axis preferred orientation of the calcite grains. Five samples (B1–B5) of a bajocian biosparitic grainstone were selected from five quarries situated on the eastern border of the Massif Central, about 15 km SW of Châlon Sur Saône (Fig. 3). In this area, the bedding plane $S_0$ dips gently to the West (N160E, 10W), and is locally affected by numerous N020E to N035E striking normal faults which are contemporaneous with the Bresse rifting. Locally, along these normal faults, fine breccias with centimetric angular blocks are cemented with white or pink clear calcite. Associated microstructures such as stylolitic peaks and subvertical N010E to N025E tension gashes are common. Although well expressed in the field, this extensional phase, probably related to the Oligocene extension of the Bresse basin, is not the only deformational phase: Two distinct sets of horizontal stylolitic peaks (respectively, NNE–SSW and NW–SE) were locally observed. The first set, classically attributed to the upper Eocene compressive “Pyrenean” phase (Bergerat, 1977), are sometimes associated with strike-slip faults (N010E to N035E, sinistral) that clearly predate the normal faults. The second set, generally poorly expressed in the studied area, may result from a Mio-Pliocene “Alpine” compressive phase related to thrusting of the Jura fold and thrust belt onto the Bresse basin (Fig. 3). For a general interpretation of the tectonics of the Bresse Basin, see Bergerat et al. (1990).

Regarded on thin section scale, bioclasts (echinoid spines, bryozoans and mollusks) are often overgrown by a syntaxial cement rim composed of a sparry calcite monocrystal. The pores between the grains were later filled with a cement composed of large euhedral transparent isodiametric calcite grains.

Some rare deformational microstructures were observed in the sample thin sections: (1) Stylolitic peaks. The peaks are oriented either perpendicular to the bedding plane $S_0$ (the most abundant) or strike N105E to N140E. (2) $e$ twin lamellae in the calcite grains. They are straight and thin (about 0.5 µm in width) and traverse the crystal. These distinctive features lead us to interpret these $e$ twin lamellae as the result of a low temperature deformation (Burkhard, 1993). In the different samples, the average twin density and the percentage of twinned crystals are very low, less than 10 per mm and 11–25% respectively. Most of the crystals (92%) are twinned on only one $e$ set. Then, the natural deformation of the samples is very low, probably lower than 0.5%.

A set of three mutually orthogonal thin sections was made to study the calcite twins orientation and the C axis fabrics into the natural sample and the experimentally deformed ones.

6. Experimental deformation

The experiments were carried out in a triaxial pressure apparatus, designed in Kiel and described by Kern (1971, 1979).

Three approximately cubic samples with between 42.5 and 45.1 mm lengths were cut (Fig. 4). Two opposite faces of the cubes were cut parallel to the stratification plane $S_0$. The initial length is determined to a precision of $10^{-3}$ mm. The six cube faces were polished and covered with a thin coat of graphite to prevent displacement parallel to these faces. The bedding plane $S_0$ is normal to the principal axis of compression $\sigma_1$. The three dry specimens were heated indirectly by the pistons; a very homogeneous distribution of temperature to within 1°C was obtained.
within the samples. The longitudinal strain $Dl/l$ is determined from the measurement of the piston displacement, using four sensors on each cube side. The estimated precision of the displacement is about $5\times 10^{-3}$ mm. The applied principal stresses, $\sigma_i$, are calculated and corrected for displacements normal to $\sigma_1$, $\sigma_2$ and $\sigma_3$. The estimated precision is about 5 MPa. The experiments are basically divided into three steps (summary of experimental conditions in Table 1).

Firstly, hydrostatic pressure is progressively increased, at room temperature, up to the chosen confining pressure $P_c$. Secondly, the samples are heated gradually, at constant confining pressure. Lastly, a load is applied at constant confining pressure and temperature (Fig. 5 and Tables 3–5).

The two samples B21 and B31 have been subjected to axial stress, with $\sigma_2 = \sigma_3 = P_c$. The sample B32 has been subjected to plane strain conditions, with $\varepsilon_2 = 0$ on $Y$. In this last sample (Fig. 6), the SESR $\Phi$ has increased irregularly at the beginning of the experiment, from 0 to 0.4 ± 0.02; then, this value was almost constant (0.43 for the peak of differential stress $(\sigma_1 - \sigma_3)$).

7. Interpretation of experimental deformation

For the three experiments, the stress/strain curves were divided into three stages (I–III) (Table 2 and Fig. 5). The comparison of the data was made possible by normalizing the strain and stress parameters for a constant 100 MPa increase of the differential stress $(\sigma_1 - \sigma_3)$.

7.1. Stage I

At the beginning of the experiments, a very linear relationship between $\varepsilon_1$ and $(\sigma_1 - \sigma_3)$ is observed. This stage corresponds to a differential stress value $(\sigma_1 - \sigma_3)$ between 0 and 80 MPa. For the three samples, the SESR $\Phi$ remains equal to 0 $(\sigma_2 = \sigma_3)$. The normalized linear strain is about 0.4% for $\varepsilon_1$ and 0.01–0.03% for $\varepsilon_2$ and $\varepsilon_3$, respectively. The normalized average volume change $\Delta V/V$ is very low: −0.3 to −0.47%.

This deformation may be interpreted as essentially elastic with a calculated Young’s modulus $E$ equal to 26.3 ± 1.5 GPa. In Fig. 5, the CRSS values for twinning on $e$ and gliding on $r$, according to Turner and Heard (1965) and De Bresser and Spiers (1997) have

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Interpretation of experimental deformation data</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: Elastic</td>
<td>II: Transition</td>
</tr>
<tr>
<td>$(0 \leq (\sigma_1 - \sigma_3) \leq 80$ MPa)</td>
<td>$(80 \leq (\sigma_1 - \sigma_3) \leq 155$ MPa)</td>
</tr>
<tr>
<td>Normalized $\varepsilon_1$ ($10^{-3}$)</td>
<td>$-3.6$ to $-4$</td>
</tr>
<tr>
<td>Strain rate ($10^{-7}$ s$^{-1}$)</td>
<td>$2.3$–$3$</td>
</tr>
<tr>
<td>Normalized $\Delta V/V$ ($10^{-3}$)</td>
<td>$-3.32$ to $-4.7$</td>
</tr>
<tr>
<td>Young’s modulus $E$ (GPa)</td>
<td>$26.3$ ± $1.5$</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>$0.03$–$0.13$</td>
</tr>
</tbody>
</table>
Fig. 5. Linear strain versus principal differential stress ($\sigma_1 - \sigma_3$) for the three samples B21, B31 and B32. I–III are the successive stages of deformation defined in the text; Dashed lines (1) correspond to the minimum values of the principal differential stress ($\sigma_1 - \sigma_3$) for twinning on $e$ at yield, and for gliding on $r$, according to Turner et al. (1954). Dashed line (2) corresponds to the minimum value of the principal differential stress ($\sigma_1 - \sigma_3$) for gliding on $r$, after De Bresser and Spiers (1997) (extrapolated from their Fig. 6). Dashed line (**) corresponds to steady-state creep during the night.
been reported (straight lines 1 and 2, respectively). It is therefore likely that twinning on the most favorable oriented calcite grains, may begin very early during the deformation process, i.e. before the end of this elastic deformation. At 300°C, gliding onto the r planes may also be possible for some well-oriented calcite grains.

7.2. Stage II

This second part is characterized by a different mean slope of the strain/stress curves (Fig. 5); it corresponds to a domain of differential stress values \( \sigma_1 - \sigma_3 \) from 80 to 155 MPa. This stage is characterized by a nearly linear relationship between \( \varepsilon_1 \) and \( \sigma_1 - \sigma_3 \) and quasi-axial stress conditions. Young’s modulus \( E \) is about half the previous one and the linear strain rate is three to four times the previously calculated one.

7.3. Stage III

The last part of the strain/stress curves corresponds to a characteristic ductile behavior. The linear deformation \( \varepsilon_1 \) increases exponentially from about \(-4.83\); \(-8.73\) and \(-9.38\)% in B21, B31 and B32, respectively.
respectively. The SESR $\Phi$ value increases from 0.2 to 0.4 (Fig. 6).

8. Inverse calculation of the RST

Tables 3–5 show the different parameters that will be used to define the best solutions. These parameters are: (1) The percentage $P$ of $e$ twin lamellae compatible with the RST. (2) The percentage of untwinned planes incompatible with the RST. (3) The maximum value ($S_{UT}$) of the resolved shear stress $\tau_i$ applied on the untwinned planes incompatible with the RST. (4) The value of the penalization function $F$. This last parameter indicates the quality of the arrangement of the whole measured $e$ planes (for details, see Lacombe and Laurent, 1996). Only the computed RST corresponding to the experimental deformation (Fig. 7) will be mentioned in this paper.

The interpretation of these parameters leads to the definition of the RST best solutions, and the acceptable RST solutions. In the three studied samples, three

Table 4

| Sample B31. Same legend as for Table 3. 145 crystals; twinned planes 342 (78.6%); untwinned planes 93 (21.4%). Interpretation: best solution is for 49% of compatible twins. a and b are acceptable solutions (for details, see text) |
|---|---|---|---|---|---|---|---|
| %$(1)$ | %$(2)$ | $S_{UT}$ | $\theta \sigma_1$ | $\Phi$ | $F$ | $\tau_i$ | CRSS |
| 45 | 0 | – | 09 | 0.037 | 0 | 0.0579 | 15.46 |
| 46 | 1 | 0.068 | 08 | 0.041 | 0.02 | 0.0521 | 13.91 |
| 47 | 2 | 0.064 | 10 | 0.014 | 0.02 | 0.0432 | 11.53 |
| 48 | 0 | – | 07 | 0.202 | 0 | 0.0391 | 14.44 |
| 49 | 1 | 0.039 | 04 | 0.202 | 0 | 0.0380 | 10.15 |
| 50 | 1 | 0.050 | 04 | 0.238 | 0.02 | 0.0337 | 9.0 |
| 51 | 2 | 0.067 | 07 | 0.108 | 0.06 | 0.0268 | 7.16 |
| 52 | 3 | 0.075 | 07 | 0.129 | 0.08 | 0.0220 | 5.87 |
| 53 | 4 | 0.049 | 04 | 0.157 | 0.10 | 0.0081 | 2.16 |
| 54 | 3 | 0.060 | 08 | 0.579 | 0.11 | – 0.0023 | * |

Table 5

| Sample B32. Same legend as for Table 3. 94 crystals; twinned planes 240 (85%); untwinned planes 42 (15%). $\theta \sigma_3$, angle in degrees between the experimental principal stress axis $\sigma_3$ and the calculated one; the theoretical $\Phi$ value is 0.43. Interpretation: best solution is for 52% of compatible twins; a and b are acceptable solutions (for details, see text) |
|---|---|---|---|---|---|---|---|
| %$(1)$ | %$(2)$ | $S_{UT}$ | $\theta \sigma_1$ | $\theta \sigma_3$ | $\Phi$ | $F$ | $\tau_i$ | CRSS |
| 47 | 0 | – | 04 | 06 | 0.312 | 0 | 0.0569 | 18.0 |
| 48 | 4.8 | 0.090 | 05 | 06 | 0.284 | 0.01 | 0.0540 | 17.1 |
| 49 | 4.8 | 0.063 | 04 | 06 | 0.301 | 0.01 | 0.0534 | 16.9 |
| 50 | 4.8 | 0.084 | 06 | 10 | 0.308 | 0.03 | 0.0519 | 16.4 |
| 51 | 7.1 | 0.098 | 11 | 09 | 0.405 | 0.09 | 0.0532 | 16.8 |
| 52 | 7.1 | 0.070 | 07 | 11 | 0.384 | 0.04 | 0.0424 | 13.4 |
| 53 | 4.8 | 0.136 | 10 | 10 | 0.398 | 0.13 | 0.0449 | 14.2 |
| 54 | 16.7 | 0.072 | 05 | 12 | 0.324 | 0.13 | 0.0241 | 7.62 |
| 55 | 11.9 | 0.079 | 07 | 09 | 0.363 | 0.16 | 0.0186 | 5.88 |
| 56 | 7.1 | 0.140 | 05 | 11 | 0.338 | 0.17 | 0.0239 | 7.55 |
| 57 | 9.5 | 0.123 | 01 | 10 | 0.283 | 0.20 | 0.0133 | 4.2 |
| 58 | 11.9 | 0.108 | 02 | 17 | 0.295 | 0.23 | 0.0145 | 4.58 |
| 59 | 11.9 | 0.111 | 02 | 16 | 0.302 | 0.24 | 0.0110 | 3.47 |
| 60 | 19 | 0.114 | 01 | 14 | 0.311 | 0.31 | – 0.0059 | * |
| 70 | 33.3 | 0.145 | 06 | 21 | 0.172 | 1.04 | – 0.0567 | * |
Fig. 7. Definition of the RST best solutions (indicated by the vertical lines BS) in the three samples B21 (left of this figure), B31 (upper right) and B32 (lower right). On the upper part, are the diagrams of $\tau_a$ versus the percentage $P$ of compatible twin planes (horizontal axis); $\tau_a$ is the minimum value of the resolved shear stress $\tau_i$ for the twinned planes compatible with the best RST solution (vertical axis on the left); on the same diagram, the percentage of incompatible untwinned planes is also drawn (vertical axis on the right). Underneath, are the diagrams of the penalization function $F$ (vertical axis) versus the percentage $P$ of compatible twin planes (horizontal axis). a–d and the shaded areas are the acceptable solutions defined in Tables 3–5.
to five RST solutions are considered to be acceptable which define the interval of confidence for the different parameters of the calculated DST.

Tables 3–5 also indicate: (1) The angle between the computed principal stress axis $\sigma_1$ ($\sigma_1$ and $\sigma_3$ in the case of B32) and the experimental one(s). (2) The computed SESR $\Phi$ values. (3) The minimum value $\tau_s$ of the resolved shear stresses $\tau_s$ applied on the twinned planes compatible with the RST (see also Fig. 7). These $\tau_s$ values are used to calculate the CRSS $\tau$ for twinning on $e$:

$$\tau_s = \tau_2(\sigma_1 - \sigma_3) \tag{2}$$

8.1. Sample B21: axial stress; $T = 201^\circ C$

In this sample 36.6% of the measured $e$ planes are untwinned. This relatively high value is probably linked to the low final linear strain $\varepsilon_i$ (less than 5%). When no preferred orientation of the calcite grains is observed, such a high percentage of untwinned planes is always correlated with a high percentage of classified incompatible untwinned planes (Table 3; column 2); then the penalization function value $F$ is much higher for B21 than for B31 and B32. The two main reasons that may account for this are: (1) The effective CRSS value which actually depends on the amount of strain (see discussion below). (2) The greater heterogeneity of strain (and stress) in the low deformed samples, due to the large mechanical anisotropy of the calcite crystals (see also Spiers, 1979).

Table 3 shows 13 trials which correspond to an increasing percentage $P$ comprised between 70 and 80%. If we reject the last line ($P = 80\%$), which is clearly a wrong solution ($\tau_s$ is negative), one may observe that: (1) The computed stress tensor orientation is defined quite precisely as $3^\circ$–$6^\circ$; it is the same order of precision as that of the making of the thin sections (particularly the sawing process); (2) the SESR $\Phi$ value is much more variable, from about 0.05 to about 0.17; (3) The $F$ value increases more or less regularly with the percentage $P$.

The best computed inverse solution is defined for $P = 72\%$. The percentage of incompatible untwinned planes is relatively high, with respect to the solutions which are close to it ($a$–$d$); this means that the arrangement of these incompatible untwinned planes is such that they are situated more on the left in the diagram of Fig. 2b (see also Fig. 8). A comparison of the two RST calculated with $P = 72\%$ (the best solution and solution c in Table 3), shows that the best solution is essentially due to a different $\Phi$ value (0.133 in place of 0.052).

Considering the best solution and the acceptable ones, the results may be summarized as follows: (1) the principal stress direction $\sigma_1$ is at $3^\circ$–$5^\circ$ from the theoretical one; (2) the SESR $\Phi$ value is equal to 0.13 ($+0.03; -0.08$); for the best solution ($\Phi = 0.133$), the calculated $\sigma_2$ value is 135 MPa, which corresponds to an overestimation of 32.5%; (3) the $\tau_s$ value is equal to 0.0606 ($+0; -0.007$); as the maximum of the applied differential stress ($\sigma_1 - \sigma_3$) was equal to 248 MPa (Table 1), the computed CRSS for twinning on $e$ (Fig. 9) is about 15 MPa ($+0; -1.6$).

8.2. Sample B31: axial stress; $T = 305^\circ C$

In sample B31, the best solution is defined for $P = 49\%$ and two acceptable solutions are found for $P = 48$ and 50% (see Table 4 and Fig. 8). (1) The principal stress direction $\sigma_1$ is at $4^\circ$–$7^\circ$ from the theoretical one. (2) The SESR $\Phi$ value is equal to 0.2 ($+0.04; -0$); 3) the $\tau_s$ value is equal to 0.038 ($+0.001; -0.005$). Then, using Eq. (2) and the actual value of $(\sigma_1 - \sigma_3)$, i.e. 267 MPa, the computed CRSS for twinning on $e$ (Fig. 9) is equal to 10.1 MPa ($+0.5; -1$).

As for sample B21, the precision of the principal stress direction $\sigma_1$ is good, but the SESR $\Phi$ value is very variable for the different trials. For the best solution ($\Phi = 0.202$), the calculated $\sigma_2$ value is 267 MPa, which corresponds to an overestimation of 25%.

8.3. Sample B32: triaxial stress; $T = 305^\circ C$

In sample B32, the best solution is defined for $P = 52\%$; and two acceptable solutions are described for $P = 49$ and 50%. (Table 5 and Fig. 8). (1) The principal stress direction $\sigma_1$ is at $4^\circ$–$7^\circ$ from the theoretical one. (2) The principal stress direction $\sigma_3$ is at $6^\circ$–$11^\circ$ from the theoretical one. (3) The SESR $\Phi$ value is equal to 0.384 ($+0; -0.08$) which corresponds to an underestimation of about 12%. (4) The $\tau_s$ value is equal to 0.0424 ($+0.011; -0$). Then, using Eq. (2) and the actual value of $(\sigma_1 - \sigma_3)$, i.e. 316 MPa, the
computed CRSS for twinning on e (see Fig. 9) is equal to 13.4 MPa (+3.5; −0).

9. Discussion of the results

The calculated principal stress orientations show very stable solutions from one RST solution to the other (Tables 3–5). They correspond within a few degrees to the experimental ones. Although this result was predictable for axial stress (Lacombe and Laurent, 1996), it is the first time that plane strain conditions are used for the inverse analysis. The results concerning the sample B32 show that the principal intermediate stress $\sigma_2$ is well determined.

In a first approach, the magnitude of the principal differential stresses $(\sigma_1 - \sigma_3)$ was calculated according to the hypothesis used up till now, that the CRSS value $\tau_s$ is constant and equals 10 MPa. For the best solutions, we obtained: 165, 263 and 236 MPa for B21, B31 and B32, respectively. Compared to the peak of applied differential stress $(\sigma_1 - \sigma_3)$ (Table 1), these values are underestimated by 33.5, 1.5 and 25.5%, respectively. If we take into account the acceptable solutions, the range of variation for the $(\sigma_1 - \sigma_3)$ values is: −33.5 to −23.8% for B21; −4 to +11% for B31; and −25.5 to −41% for B32. Thus, these three experiments carried out at 200 and 300°C lead to a general underestimation of the $(\sigma_1 - \sigma_3)$ values by about 40%.

The calculation of the two differential stress magnitudes $(\sigma_2 - \sigma_3)$ and $(\sigma_1 - \sigma_2)$, from Eq. (2) and the SESR $\Phi$ value shows that: (1) For axial stress ($\Phi = 0$), the results are rather unstable (Tables 3 and 4). This is probably related to the fact that the minimization process of the Etchecopar inverse method is mainly dependent on the incompatible untwinned planes; consequently, the orientation fabrics of the untwinned planes must influence these results. (2) For triaxial stress, the results are very close to the expected ones (Table 5). As it is the first time that experimental triaxial stress is analyzed using an

Fig. 8. Classification of the whole set of e planes (twinned and untwinned) with respect to the applied resolved shear stress $\tau_i$, computed from the best RST solutions for the samples B21, B31 and B32. On the horizontal axes, are the resolved shear stress $\tau_i$ values, in the range −0.5 to +0.5 from left to right of the diagram. On the vertical axes, are the percentages of classified e planes (continuous curves, twinned planes; dashed curves, untwinned planes). $\tau_a$ is the minimum value of the resolved shear stress $\tau_i$ for the whole set of compatible twin planes. Percentages of compatible twinned planes (upper horizontal lines) and incompatible untwinned planes (lower horizontal lines) are indicated for the three samples.
inverse method, further data are needed to draw definite conclusions because, as in the previous cases, the fabrics of incompatible untwinned planes should influence this result.

An alternative use of these experimental data has been the calculation of the CRSS values $\tau_s$ for twinning on $e$ from Eq. (2) and the actual values of $(\sigma_1 - \sigma_3)$. These $\tau_s$ values have been plotted in Fig. 9, together with those of Turner et al. (1954), Turner and Heard (1965) and Lacombe and Laurent (1996). For each sample, the percentage of strain is indicated. The continuous line of the Fig. 9 is defined at yield, which probably corresponds to the maximum shearing stresses at which twinning is initiated (Carter, 1976). The dashed line of the Fig. 9 is calculated for 3% of finite strain. These two curves show that the CRSS values $\tau_s$ for twinning on $e$ are strain dependent. That strain dependence is linked to the fact that twinning is very sensitive to the presence of inhomogeneities within the crystals (Schmid and Boas, 1950). Thus, during the deformation process, twinning must get more and more difficult with the increase of strain and the resolved shear stress value necessary to twin the worst oriented $e$ planes (i.e. the value which defines the CRSS value $\tau_s$ for twinning on $e$), will increase with strain. This may explain the fact that in the higher deformed samples the principal differential stress values $(\sigma_1 - \sigma_3)$, calculated using a constant 10 MPa value for $\tau_s$, are generally underestimated. In our deformed samples, twinning has been achieved probably as early as the first stage of deformation (I in Fig. 6), within the calcite grains which were the most favorably oriented. Then, during stages II and III, twinning on $e$ may have developed into more and more calcite grains, even when the gliding on the $r$ planes should have become an easier mechanism for the deformation (Fig. 5).
The consequences of these results on the determination of principal differential stress values for geological cases, particularly in the foreland of mountain chains (e.g. Lacombe and Laurent, 1991, 1996; Craddock et al., 1993; Van Der Pluijm et al., 1997) may be summarized as follows: for the low strained sparitic limestones (i.e. strain lower than about 1.5%), the CRSS value \( \tau_s \) for twinning on \( e \) which may be used is more likely about that proposed by Turner et al. (1954) for incipient twinning: at room temperature, it is about 5–7 MPa. As a result, the previous determinations of \( \sigma_1, \sigma_2, \sigma_3 \)† in the foreland of the Pyrenean chain (Tourneret and Laurent, 1990; Lacombe et al., 1991, 1996; Lacombe and Laurent, 1992) may have been overestimated by a factor in the range 1.43–2.

10. Conclusions

This experimental work confirms that: (1) inverse determination of DST gives reliable results for the principal stress orientations, even when the state of stress is triaxial; (2) the CRSS values \( \tau_s \) for twinning on \( e \) in calcite grain are mainly dependent on both temperature and strain (an increase of temperature/strain respectively lowers/increases the \( \tau_s \) values). For limestones strained to about 5–10%, the \( \tau_s \) values are circa 10–15 Mpa at 200 and 300°C.

On the other hand, this work confirms that the \( \tau_s \) value proposed by Turner et al. (1954) for incipient twinning, i.e. 5–7 MPa, is more likely for slightly deformed limestones.

The use of the Etchecopar method (1984) together with the correct values of \( \tau_s \) defined in Fig. 9, should lead to the magnitude of \( (\sigma_1 - \sigma_3) \) with an interval of confidence estimated at about 50%. This is much better than the overestimation by a factor up to 20, obtained with the methods of Jamison and Spang (1976) and Rowe and Rutter (1990) by Ferrill (1998). In order to confirm these results for the application to the paleostress fields associated with the tectonics of mountain forelands, further experiments, particularly at low temperatures (0–200°C) and low strain (0–2%), are needed to support these results.

Acknowledgements

This work was supported by a “programme d’actions intégrées” PROCOPE. We wish to thank J.P. Burg from the ETH Zürich, for the improvement of this manuscript. Special thanks go to Dana Petit for the revision of the English version, to Anne Delplanque for the drawings and M. Peret for help in data processing. K. Burmeister prepared the samples and M. Schroetel assisted in performing the experiments. The thin sections were made by L. Cammal.

References

Engelder, T., Geiser, P., 1980. On the use of regional joint sets as trajectories of paleostress fields during the development of the


Lacombe, O., Angelier, J., Laurent, Ph., Bergerat, F., Tourneret, Ch., 1990b. Joint analyses of calcite twins and fault slips as a key for deciphering polyphase tectonics: Burgundy as a case sample. Tectonophysics 182, 279–300.


